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TABLE XXIII.—SUMMARY of the Returns from the following Departments, viz.:—*Treasury, Customs, Excise, Post Office, Ordnance Office, Stamps and Taxes, War Office, Colonial Office, Public Record Office, Paymaster General's Office, National Debt Office, Admiralty, General Register Office, and Stationery Office.*

Officers' Salaries. £	Ages of Officers.												
	Total	15.	20.	25.	30.	35.	40.	45.	50.	55.	60.	70 and upwards.	Not specified.
Total	7964	149	686	1455	1629	1315	665	539	485	479	483	58	21
Day Pay & under 50	1681	108	294	393	328	152	103	87	85	115	14	2
Day Pay & 50	1273	54	154	261	299	214	81	45	41	40	75	7	2
80	520	47	128	118	66	67	27	20	15	11	17	4
90	391	42	89	90	45	36	30	16	7	16	13	3	4
100	1668	6	102	390	467	338	118	111	61	34	31	7	3
101	62	2	1	4	1	2	5	11	26	10
110	151	22	28	26	27	18	5	5	4	16
120	199	25	51	27	26	20	15	10	12	10	2	1
130	102	6	22	23	14	3	7	13	7	6	1
140	94	7	22	15	17	11	7	8	3	4
150	514	35	107	119	75	36	49	34	36	20	2	1
175	110	1	16	15	13	8	13	10	16	17	1
200	430	2	36	71	71	67	51	54	43	29	5	1
225	46	4	3	5	5	11	1	3	8	6
250	222	9	29	35	27	22	29	29	36	5	1
300	181	1	4	11	26	18	24	26	38	27	4	2
350	80	2	9	7	12	14	14	16	5	1
400	89	4	8	11	15	19	16	15	1
450	33	1	2	4	13	9	4
500	54	5	4	5	12	13	12	3
600	22	1	4	1	8	5	3
700 & upds	30	1	1	4	3	11	10
Not specified	12	1	2	2	2	1	2	1	1

This table may be read thus:—Out of 7,964 officers who made returns, 514 are receiving 150*l.*, and under 175*l.* per annum; 35 of the number are 20 and under 25 years of age, 49 are 45 and under 50 years of age.

It will be recollected, as explained in the remarks, that only 7,964 out of 11,500 officers in the respective departments made returns; and that the senior officers and higher salaries are not in due proportion. A complete table may, however, be formed on this plan by simply requiring, through the heads of departments, a return of the ages and salaries of the several officers.

On Funds to provide for the Fatherless Children and Widows of Civil Officers—or of others—from a Deduction out of their Salaries.

Children, women, and men advanced in years, can rarely earn enough to supply themselves with subsistence. Men who enter a profession have, therefore, during their years of active life, not only to supply the current wants of their families, but to make a provision for the infirmities of sickness or age, and in consequence of the mortality of their nature, for their widows and children. The annexed table shows that in six states of Europe there are to every *four* wives *one* widow—and an unknown number of fatherless children, which the returns leave undetermined. The children of the possessors

Female Population of Eight Nations, distinguishing Wives, Widows, and Spinsters, aged 15 Years and upwards.

	Total Female Population.	Female Population aged 15 Years and upwards.	Wives.	Widows.	Spinsters aged 15 Years and upwards.	Proportion per Cent. to Female Population aged 15 years and upwards, of				Proportion per Cent. to Female Population. All Ages.		Proportion per Cent. of Females, aged 15 Years and upwards, to Female Population, all ages.
						Wives.	Widows.	Spinsters aged 15 Years and upwards.	Wives.	Widows.		
1. Sweden (1835).....	1,563,862	1,032,377	513,087	145,460	373,830	49,700	14,090	36,211	32,809	9,301	66.015	
2. Norway (1835)	609,446	401,672	193,058	42,660	165,954	48,064	10,621	41,316	31,678	7,000	65.908	
3. Hanover (1842)	882,248	586,035	279,876	74,325	231,834	47,758	12,683	39,560	31,723	8,425	66.424	
4. Saxony (1840)	876,621	583,916	299,191	69,440	215,285	51,239	11,892	36,869	34,130	7,921	66.610	
5. Holland (1840)	1,362,410	905,149	415,374	113,401	376,374	45,890	12,528	41,581	30,488	8,324	66.437	
6. Ireland (1841)	*4,152,071	2,526,183	1,181,095	312,420	1,032,668	46,754	12,367	40,879	28,446	7,524	60.842	
7. Prussia (1840)	7,479,919	4,764,377	2,474,177	2,290,200	2,290,200	51,931	48,069	48,069	33,078	63.696	
8. France (1841)	17,321,504	6,461,521	1,608,645	37,303	9,287	
Sum of Nos. 1-8	34,248,081	11,817,379	34,505	
" " 1-7	16,926,577	10,799,709	5,355,858	49,593	31,642	63.803	
" " 1-6	9,446,658	6,035,332	2,881,681	757,706	2,395,945	47,747	12,555	39,699	30,505	8,021	63.889	
" " 1-6, 8, } viz., Prussia excluded }	26,768,162	9,343,202	2,366,351	34,904	8,840	

* 3,477 females, whose ages were not specified, are excluded from this Return.

of property are naturally provided for under our laws ; and the children of the man whose income is derived from his industry generally enjoy the same privilege ; but the source of income depending on his life, they are liable at any time to be thrown on the community for support, which is in a high degree precarious. In the middle and higher classes they are practically thrown upon the hands of their relatives—of the charitable—and, in some rare instances, of the parish. Life insurance meets the risk of mortality ; but it unfortunately happens in all professions—and in the civil service among others—that life insurance, to an adequate extent, is not effected by the great majority of husbands—and more particularly by those whose lives are most liable to be cut short, and whose large families are likely to prove the severest pressure of want—the heaviest burthen on the community. Society has, therefore, a right, and whenever an opportunity offers, perhaps a duty to see, that such a deduction is made from the adequate income in active life as will lighten the sufferings of the fatherless children and widows of its members. If the Government set the example in the public service, it may be copied by other classes ; and would ultimately prove a great boon and economy to the nation.

The following Table (p. 128) shows the constitution of the families of civil servants. A few enter the service under the age of 20. It will be seen that 100 officers of that age (15—20) had 77 fathers, 90 mothers, and 209 sisters living. At the age (25—30) of 100 many are married; 64 have fathers, 72 mothers, 49 wives, of whom 35 have children, 14 no children, and the children living are 63 ; 222 sisters of the 100 officers are living. Advancing to the age (50—55) the constitution of the family has undergone great changes : 100 officers have only 8 fathers, 13 mothers, 144 sisters living ; and 83 wives, of whom 67 have children, 16 no children ; the children are 351, of whom 131 are under, 220 are above, 15 years of age. Following the man in his procession through life, the *number* of the family grouped around him is greatest (9) at the ages 30—50 ; but it differs little all along from the age of 25 to 65 ; as the fathers, mothers, sisters and brothers, on whom he rested silently disappear, the children take their place, and rest upon him. After threescore years and ten his children decrease, and he would appear lonely, if the grand-children, which our table omits, did not take their place, and shed verdure and youth round his decay. The Table, which is, perhaps, one of the most curious that has been framed, presents many points of practical interest. It will be observed that, besides wives and children, officers may have fathers, mothers, sisters, and other relatives dependent on them for support.

Proportion of Fathers, Mothers, Sisters, Wives, and Children to each Civil Officer, at Thirteen Periods of Life.

Ages.	Fathers of Officers.	Mothers of Officers.	Sisters of Officers.	Officers.	Wives			Children				Total Numbers of Members of Family.	Officers who made Returns.
					Total.	having Children.	not having Children.	Total.	under 15 years of age.	above 15 years of age.	to Officers having wives and children.		
15	·772	·899	2·093	1·000	6·857	149
20	·726	·775	2·298	1·000	·183	·112	·071	·152	·152	...	1·364	7·432	689
25	·635	·715	2·220	1·000	·487	·351	·135	·629	·629	...	1·791	7·906	1,455
30*	·533	·603	2·021	1·000	·734	·581	·153	1·578	1·572	·006	2·717	8·490	1,629
35	·396	·496	1·963	1·000	·820	·669	·151	2·406	2·300	·106	3·599	9·047	1,316
40	·230	·410	1·691	1·000	·846	·714	·133	2·956	2·331	·625	4·141	8·824	664
45	·189	·283	1·631	1·000	·865	·717	·148	3·417	2·113	1·304	4·767	9·016	540
50	·080	·134	1·436	1·000	·829	·674	·155	3·511	1·311	2·200	5·208	8·427	485
55	·030	·059	1·354	1·000	·827	·700	·127	4·110	·960	3·150	5·867	8·734	474
60	·016	·032	1·134	1·000	·818	·666	·153	3·914	·449	3·465	5·880	8·048	314
65	·024	·868	1·000	·772	·647	·126	3·964	·419	3·545	6·130	7·496	167
70	·722	1·000	·759	·630	·130	3·352	·167	3·185	5·324	6·555	54
75	1·000	·286	·286	...	2·571	...	2·571	9·000	3·857	7

This Table may be read thus, without reference to the decimal points:—Of 1,000 Officers, at the age of 50 and under 55, 829 have wives living; 674 of whom have children living; the children are 3,511, or 5·208 (nearly $5\frac{1}{2}$) to each wife. And so of other ages.

* It will be observed that in the original return there is an undue proportion of persons between the ages of 30 and 40. In constructing the 13th column it is assumed that the number of brothers is equal to the number of sisters.

The primary object of the Civil Service Fund is to secure ultimately a certain provision for the widows, orphans, and dependent relatives of the lowest, as well as the highest, officers in all branches of the civil service; and that exclusively from the tax of $2\frac{1}{2}$ or 5 per cent. deducted from their salaries under 4th and 5th Will. IV, cap. 24. I do not enter at all into the machinery or the plan proposed for the management of this Fund; on which the very able Report of the Committee contains ample information.

The scales of offices differ; but in general the salary rises with length of service; and as promotion often goes by merit, salaries under the present imperfect scales bear some proportion to efficiency and length of service. The contribution is in a fixed proportion to the salary; therefore, in proposing that the benefits shall be equivalent to the contributions, we make the provision for the widows and orphans depend to a great extent on the conduct and services of the officer.

First Case.—For an illustration we will take the case of an officer B, who enters a public office at the age of 17. His salary in the *sixth* year of service is 120*l.*; his contribution 6*l.* a-year; his claim on the fund at death is 390*l.*; he rises by seniority alone, and his salary in the *twentieth* year of service is 260*l.*; his contribution in the same year is 13*l.*; his claim on the fund at death is 727*l.*

That is the sum for which his several contributions would insure his life. If he were married the widow's claim would vary, at the two periods, in certain proportions to 390*l.* and to 727*l.*

Second Case.—C enters the same office at the same age. In the *sixth* year of service his salary is 120*l.*; his annual contribution 6*l.*; his claim on the fund at death, whenever that may take place, provided the contribution be continued, is 390*l.*; in the *twentieth* year of service his salary, through a more rapid promotion than B has obtained, is 600*l.*; his annual contribution is 30*l.*; his claim on the fund is 1,538*l.*

That is the average sum to which the *contributions* of men of his age, put out at 3 per cent. interest, would amount if the duration of their lives were equal to that of the general population of the country.

Here the claims on the fund of both B and C rise with length of service. If B should complain, it will be with the salary and not with the principle, which makes his claim on the fund equivalent to the value of his contributions.

The funds which for ten or twenty years' service grant the same survivorship annuity on the life of an old or young widow of an old or young officer, giving for services presumed to be of the same value, very different rewards, are unjust; and they are liable to this serious objection, that it is impossible to foresee the extent to which the claims will go; or to value at any time the assets and liabilities of the fund.

The practice is to be deprecated, for the same reason, of funds which levy a per centage on the salaries and allowances of single men, and of men who marry at advanced ages, for the almost exclusive advantage of married men, and more particularly of officers who marry young.

There is no justice in this practice; and the funds are always insecure, as it is impossible to calculate beforehand how many members will or will not marry, at what ages they will marry, and how many claimants will fall within any given time.

The existing institutions, of which a valuable analysis has been made by Mr. Willis, of the Admiralty, afford us many useful suggestions. The chief errors to avoid in them appear to be:—1. The confusion of benevolent but purely arbitrary arrangements with transactions depending on the duration of life, and admitting of strict calculation; 2, complicated regulations, arising out of the vain attempt to define every case that may arise; and, 3rdly, the inequitable distribution of the benefits without a strict regard to the just claims of the contributors. All these errors will be avoided by adhering to the principle of equitable insurance, and keeping the accounts of the benevolent fund entirely apart from those of the provident fund. The capital of the provident fund will be the contributor's property, of which the directors will be the trustees, for the benefit of his widow and children if the officer be married; the operations of the benevolent fund will be free, and at the disposal of the directors, to mitigate the severity of affliction or poverty in the families of poor officers—to foster their fatherless children till they are ready to engage in the business of the world.

The following remarks apply exclusively to the "provident" fund; and I shall lay down these principles for our guidance. 1. The benefits to each contributor are to be as nearly as possible equivalent to his contributions. 2. The contributor is to enjoy the advantages of life-insurance; so that should his decease take place on the day after he has been enrolled on the list among the claimants on the fund, and has paid 10*l.*, he will leave the same sum to purchase annuities for his widow and children as a man of the same age who lives to contribute 10*l.* a-year for thirty years. It being assumed that contributors of the same age have the same *chances* of living any number of years; and that, therefore, their wives and children have a right to the same sum; or, which is the same thing, to annuities of the same value. 3. That as it is the duty of every married man to provide for his wife and children, in the event of death, the claims of the widow and orphans are to supersede all others. 4. But that, these being provided for, no more control is to be exercised over the distribution of the contributor's property invested in this fund, than is clearly for the good of his family. It will become, under these arrangements, the interest of many officers of long standing, not subject to the tax, to contribute to the fund; as, for the same deposits, they will nowhere else be able to obtain the same advantages for their families. 5. If Her Majesty's Government should be pleased to sanction the establishment of the fund, the Treasury should have ready means of ascertaining its liabilities at any time. The elementary principles of Life Insurance are so generally understood in the civil service, that it is unnecessary to discuss them. It will only be necessary to advert to the many important particulars in which the *operations* of the "Civil Service Fund" will differ from the ordinary transactions of insurance. If a young man at the age of 20 wishes to insure his life, he first fixes on the sum to be insured. Say it is 1000*l.* On going to a Life Office he finds the uniform annual premium required by the common tables to be 21·790*l.** By paying 21*l.* 15*s.* 10*d.*

* By the English Life Table the premium is 15·967*l.* But this includes no charge for expenses or profits.

a-year as long as he lives he is insured for 1000*l.* The *premium* varies with *the age* at which the policy is *granted**. Under the Superannuation Act—which it is proposed to disturb as little as is possible—the deductions, whatever may be the officer's age, are $2\frac{1}{2}$ per cent. where the salary does not exceed 100*l.*, and 5 per cent. from all higher salaries. Our tables must, therefore, show the sum which the payment of 1*l.* a-year, commenced at any age, and continued through life, will insure (p. 146). From this the sum which any other annual premium will insure is immediately deduced. Thus, *A*, aged 20, has a salary of 80*l.*; his contribution is 2*l.*; and as 1*l.* will insure 62·631*l.*, 2*l.* will evidently insure 125·262*l.* That is the sum which 2*l.* a-year, contributed by a large body of men, and put out at 3 per cent. compound interest, convertible annually, would pay on an average at their deaths. Salaries, however, generally rise; and *A*'s contribution to the fund in some offices would be say 2·25*l.*; 2·50*l.*; 5·5*l.*; 6*l.*, &c., &c., in successive years. This would be equivalent to effecting new insurances with the *annual premiums* of five shillings at the age, for instance, of 21, five shillings at 22, 3*l.* at 23, ten shillings at 24, &c., &c. Again, it is proposed that *A* shall not participate in the benefits of the fund until he has contributed five years. This will exclude bad lives, that may be otherwise thrown on the fund, and will prevent engagements to men who remain but temporarily in the service. After *A* has contributed some time he may marry; the fund is to provide an annuity for his widow. His wife may die before him; the fund places a sum at his disposal for his parents or sisters. He may have one, two, or ten children; the fund is to provide annuities for each of them. The child, or children, may die; the boys may live over the age of 18, the girls over 21; the widow is then to enjoy all the advantages. An officer, through infirmity, or sickness, may be incapacitated for active service, and be superannuated at any age; in which case his salary and contribution cease simultaneously. The premium being discontinued, the prospective annuity to the widow and children has to be proportionably reduced. Again, we have to deal with three classes of officers: those who in future enter the service, those now contributing, and officers in the service before 1830, who may voluntarily place themselves on the fund.

Notwithstanding all these complications in the circumstances of the several cases, the accounts of the fund may, I think, be kept clear by adopting some such plan as the following. A register is to be kept of all cases at the office, specifying the required particulars in the annexed form (A)†. When an officer has been appointed, a corresponding schedule, properly filled up, must be transmitted at the end of the quarter, with the contribution of the quarter, to the office of the fund. The contribution of the next quarter will be transmitted

* Sums of money are throughout expressed in pounds and decimals of a pound. 21·790*l.* may be read in short thus, 21*l.*, 7 decs., 9 cents. The decima, or tenth part of a pound, is = 2*s.*, and 5 cents make a shilling, the unit figure in the third decimal place is a little less than a farthing; 25 are = 6*d.* It may be called a millesima, or, in short, a *mil.* The premium by the English table is 15*l.*, 9 decs., 6 cents, 7 mils. In all tables of annuity the pound is taken for the unit.

† Copies of the Schedules are deposited in the library of the Statistical Society. It was not thought necessary to print them all as the tables are already numerous.

in the same way. The accounts of these contributions, as well as of the investments and of the payments, will be kept in the usual way. On the 1st of January, or as soon as possible after that date, the officer's name, age, salary, and contribution, will be entered in the *Insurance Account* under his department and number*. If he enter the service on July 1, 1848, for example, on a salary of 90*l.* a year, and his age, at the previous birthday, be 17, his contribution for the half-year will be 1·125*l.* (1*l.* 2*s.* 6*d.*); which will be entered in the proper column as 1·125*l.*, against January 1, 1849. His age will then be assumed to be eighteen. Let the scale of the office be such that the salary in 1849 is 90*l.*, the contribution will be 2·250*l.*, and in January, 1850, this will be entered to his account; and so on up to January, 1853, when his fifth annual contribution has been made. The sum of the five contributions, say 13·625*l.* (13*l.* 12*s.* 6*d.*), will then be taken, and entered in the column for "premiums." The capital sum this single premium will insure is, at his age (22), 37·300*l.*, which will now be entered in the proper column. In the next year he will contribute 6*l.*: that contribution, it may be assumed for the moment, will be continued through life; it may be called, for the sake of convenience, a premium, and if paid annually will insure 349·290*l.*; this, with the 37·300*l.*, will make 386·590*l.*, the aggregate amount of his claim on the fund. His contribution next year is 6·500*l.*; ten shillings more are therefore disposable, and will be paid yearly; this is a second annual life premium, commenced at 24, and will entitle him to 28·393*l.* at death. His claim on the fund in that event is 414·983*l.* On referring to the example (p. 148), this will be plain in the tabular form.

If the fund had been in existence, the officer there referred to, who entered in 1814, at the age of 17, would, in 1838, have had a claim of 798·359*l.*, and as his salary remained 300*l.*, his contribution 15*l.*, his claim would have been the same in January, 1847. At the age of 24 this officer married a lady of the age of 35, in the year following he had a child, and ultimately 4 children surviving in 1847, when he was 50. How should the fund deal with this and similar cases?

Probably in one of the three following ways:—

First Plan.—The simplest way, and that which would be intelligible to every one, would be to lay down the rule that: (1.) "The sum for which an officer who marries has a claim at death on the fund, shall, in that event taking place, when he has no children entitled to benefit, be converted into an annuity on the life of his widow†." This officer, in the service 6½ years, has a claim on the fund of 414·983*l.*, it will purchase an annuity of 21·615*l.* for his widow (11 years older than himself), should he die in the next year; the first payment being made at the end of the year in which his death happens. If he should die at the age of 52, when his 35th payment has been made, the sum of 798·359*l.*, for which he has then a claim, will entitle his wife to an annuity of 79·118*l.*, first payable at the end of the year in which her husband dies. The wife, in this case, is much older than the husband, and the annuity is higher than when the ages are equal. On the other hand the annuity of a young wife is lower, as she is likely to live longer.

* See Schedule B and Table, p. 148.

† See Table D. See also Table, p. 150.

And secondly, "that if there be children, only half, or some other proportion, of the sum claimed should go to provide the annuity for the widow, the other half being distributed among the orphans, in equal annuities, till the boys attained the age of 18, the girls the age of 21." In the case now before us, if the officer had died after his 19th payment, when his claim was 703·9161*l.*, the one-half would purchase an annuity of 22·071*l.* for the wife (age 48), the other half an annuity of 10·049*l.* for each of the four children. As the mother would be the natural guardian of the children she would receive 62·267*l.* a year for herself and children; 10·049*l.* would drop at the death or majority age (18 or 21) of each child. If she died, the children's annuities alone would remain; if she married again, her annuity would be converted into an annuity on the children.

This arrangement would allow of the cases of bachelors, married men without children, married men with children, widowers with children, widowers without children, widowers remarrying, being kept in the books of the society, on a system intelligible to everybody. If the wife should die before the officer, no part whatever of his contributions would be forfeited; the money would remain to be divided, at his decease, in annuities to the surviving children, or to his widow, if he married again. The annuity to the widow would increase for every year of service. See table, p. 150, col. 7.

The objections that may be urged against this plan are, that the annuity to the widow is too small, and that particularly if the officer die young.

Second Plan.—By another arrangement a certain sum may remain insured, and on the officer's marrying, say *half of his future contributions* may be devoted to provide an annuity for the widow. An equivalent sum would be written off from his claim on the fund. A portion of the past contributions may be applied to the same purpose. The annuity to the widow varies with (1,) the *claim*; with (2,) *her age*; and (3,) the *age of the officer at the time the arrangement is made**. The premiums are only paid during the life of both husband and wife; the forfeiture of those already paid, if the wife die before the husband, makes the annuity higher than it is on the first plan. The premium for the Survivorship Annuity ceases, however, to be demanded at the same time; which makes the annuity lower than on the third plan, where the premium is taken after the wife's death, to the end of the husband's life.

(See *Examples of Annuities on this Plan*, pp. 149-50.)

One table, (p. 148,) shows how the account may be kept on the second

* Let $\frac{S}{2}$ = half the sum for which officer age x , had a claim the year previous to his marriage, then if he married a wife age y , the survivorship annuity which $\frac{S}{2}$ would purchase would be $= \frac{S}{2} \times \frac{Mx}{Nx} \times \frac{1 + Ax, y}{Ay - Ax, y}$.

See Reg. Gen. Sixth Annual Report, Appendix. All the data of the calculations, given merely as illustrations, are from the English Life Table, the construction of which is there explained. In the same Report the symbols of the several formulae are explained.—See also Appendix to this paper, p. 142.

plan, which in relinquishing the premium at the death of the wife, follows the practice of offices that grant survivorship annuities. It is obvious that offices would have a difficulty to get the premiums after the death of the wife; which is not, however, the only reason why this course is adopted.

Another table (p. 150) presents a comparative view of the three plans.

Third Plan.—An officer at his marriage may devote the whole, or a part of his past and his future contributions (exclusive, however, of those *additional* contributions from rise of salary,) to provide an annuity for his widow.

Under such an arrangement the whole of such contributions, past and future, would be forfeited on the death of the present wife, whether it happened during the officer's life or after his death. The annuity of the widow would be augmented in proportion. Thus, under this arrangement, a man at 24, contributing 6·500*l.* a year, appropriated exclusively to the annuity, may leave his wife, of the *same age as himself*, an annuity of 39*l.* She would be entitled to this annuity if *he died*, the week after the arrangement; but if *she died*, the contribution of 6·500*l.* a year for his life would be required to pay the annuities of the widows of deceased officers who insured on the same terms as himself. Without running this risk of forfeiture the annuity of the widow would be only 20*l.* If half the contributions went to the survivorship annuities on wives of the *same ages* as their husbands, in the two cases their annuities would be respectively 19·6*l.* and 10*l.*, the rest would be for them and their children.

At 41 an officer has a claim for 798·359*l.*, his contribution is 15*l.* a year; half the past and future contributions would secure his wife, of the same age as himself, 40·433*l.* a year, so long as she survived him.

At *his* death the annuity would commence, at her death it would cease. If she died *before him* his claim to the equivalent of 399·179*l.* would lapse. The residue of the claim, in the case here supposed, 399·179*l.*, would remain to be distributed among the children at his death.

On either plan, in the event of the officer dying *after his wife*, small annuities will be provided for the young children*.

The second or intermediate plan will probably be the most desirable *on the whole*, but there are cases in which I consider the others more advantageous to the parties concerned.

It is easy to say what sum certain contributions will insure, but it is not easy to lay down any strict rule, which will distribute the fund among the wives and children of officers in the manner most advantageous. Many officers, with a full knowledge of their own circumstances, and the circumstances of their families, find it no easy matter to make a will satisfactory to themselves and equitable, and often find it necessary to alter their intentions; how then can we hope, by any invariable rule, to dispose satisfactorily of the contributions among the families of officers?

In the event of superannuation, a man will cease to contribute: and as the claim in the books is calculated on the assumption that the "premiums" will be paid at the beginning of *every year*, including that of the officer's decease, the claim in these cases must be diminished

* For specimens of the three plans see Tables C, D, E, and p. 150.

in proportion to the contributions withheld. Thus, an officer, age 41, has a claim of 798*l.* if he contribute 15*l.* a year to the end of his life; but he is superannuated at 65, and 15*l.* a year at 65 insures 191*l.*, at death; this sum subtracted leaves his claim 607*l.* A corresponding reduction must be made in the widow's annuity.

An officer superannuated should always have the option of continuing his contribution, which might be deducted from his superannuation allowance. If he were labouring under any fatal disease it would be the interest of his family to continue the contribution*.

I have shown how civil servants, appointed after the establishment of the fund, may be dealt with. Those now in the service, and contributing, may be placed on the fund at once, by some such arrangement as the following. The names, ages, contributions, &c., of every man would be returned from the several offices, and entered in the register. In the insurance account every man's claim, having been computed, would be entered as in the examples. A few of the claims, and the families, are given in the Form (F).

If the Government will transmit the contributions of 1849 to the receiver-general, this will do more than pay all the claims lapsing in 1850, supposing the lives were all insured on January 1st, 1850; and will leave a large surplus, as the fund, in the majority of cases of decease, will have only granted *annuities* to the widows and children.

The officers not contributing will have the option of joining. If, like those *contributing*, they all joined in a body, no proof of health would be required; for it may be assumed that they are all in average health. As entry will be entirely voluntary, the usual proofs of health will be required, to prevent the admission of an undue proportion of bad lives.

The annuities to the widows of officers who enter the service will not be considerable; they will not exceed an eighth or a ninth part—or if the value of the whole contribution went to the widow—a fourth or fifth part of the salary. The widows of officers *now in the service, entering on the fund later in life, will obtain still smaller annuities*; unless Her Majesty's Government should consent to allow officers the advantage of their past contributions. If the mere sum of the past contribution, without reckoning interest, were appropriated to increase the survivorship annuities of widows, it would be a great boon to many families, and but a slight, distant drain on the Treasury.

The fund, established in 1744, by Act of Parliament, among the ministers and professors in Scotland, for making provision for their widows and orphans, was one of the first of the kind, and served as the model for several others, of which Dr. Price has given an account†. It was of this nature: "For an annual payment, which began immediately, of 5 *guineas*, from 1011 contributors, 667 of whom are married persons, besides a tax on weddings, producing about 142*l.* per annum; it entitles every widow to an annuity of 20*l.* during

* An example will be found further on (p. 145,) of the effects of discontinuing the contribution. From this example it will be observed that the cessation of the contribution reduces the annuity to a greater extent than it does the sum insured.

† Works by Morgan, 7th ed., vol. i., pp. 72—137.

widowhood, and also every family of children that shall be left by such members as die without leaving widows, to 200l."

The London Annuity Society, established in 1765, for 5 guineas a year, granted a title to an annuity of 20l. to every widow during widowhood, *if the husband lived one year after admission*; of 30l., if the husband lived 7 years; and of 40l. if he lived 15 years. The fine on admission was 5 guineas under the age of 45, and 5 guineas a year, "extraordinary," for every year over 45 up to 55.

The *Laudable Society* differed little from the London; and both of them were on the brink of ruin when Dr. Price wrote*. The Scottish Fund prospered.

It will be observed in the "Tabular Abstracts of the Regulations of existing Societies," that the Fund of the East India Company's home establishment partakes most of the nature of our *third* plan. 2 per cent., for the Widows' and Orphans' Fund, is deducted from the salaries and retiring allowances of *married* men, *not only* during the *wife's* life but after her *death*, and from the salaries of *single* men. The Company also grants the fund 4,600l. a year. The Trinity House Fund proceeds on the same principle; and takes $2\frac{1}{2}$ per cent. indiscriminately from the salaries of bachelors, widowers, and married men, to provide annuities for the widows and orphans.

Under the plans previously proposed the widow and children of an officer who marries late will enjoy all the benefits of his previous contributions; which cannot therefore go, as in other funds, to augment the allowances to the widows of officers who marry and die young. The widow's annuity, it has been already remarked, will be small; not more than a *fifth part of the officer's salary*, if the children's portion be added to hers. The position of a woman left with children to live on an annuity which will barely pay the rent of the house in which she is dwelling, must be painful; yet it is to be feared that, on the present scales of salary, the allowance from the fund will, in many cases, be all that is left for the family. Should this be the case, it may no doubt be thought right, by some, to pay only a half, or two-thirds of the claim of a single officer to his sisters or parents, leaving the other part to go to a residue fund, the surplus of which may be appropriated in augmentation of the annuities; and such a proposition would be defended, on the ground that, at entry in early life, all officers have nearly an equal chance of leaving claimants on the fund. It will, therefore, be proper to show, from the returns in preparation, some approximation to the income that may be derived from this source.

In all insurance offices and in all provident "funds," conducted with prudence, an excess of premium is charged to cover expenses of management, to provide for the interest and repayment of the capital sunk in establishing them, and to guard against the possible fluctuations in the *interest of money* and the *rate of mortality*.

Our examples and tables, however, show only what benefits would be provided according to the rate of mortality prevailing among the population of England generally.

* See Dr. Price's paper, and his observations on The Best Schemes for Providing Annuities for Widows, none of which are, however, applicable to the Civil Service Fund.

The calculations are from the first English life table, as it stands in the Registrar-General's sixth report, reckoning interest of money at 3 per cent. With a proper machinery the expenses of management cannot be considerable; and Her Majesty's Government would probably take these upon itself. The capital sunk would be nothing, and the risk, therefore, of its loss nothing. The principle of *uniform premiums* through life adopted in the calculations would, with the great numbers insured, be an adequate provision against mere *fluctuations* in the mortality. The interest of money, fixed so low as 3 per cent., will make the fund secure on this side; for money, properly invested in Government securities only, is likely to yield a higher interest than 3 per cent. The annual rate of interest on consols was 3·783*l*.* in the 94 years, 1731-1824, and only fluctuated from 6*l*. to 2·975*l*. per cent. The nature of the fund would prevent losses from the forced purchase or sale of stock, as the payments into and out of it would, financially considered, be life annuities, and be distributed, therefore, over a great number of years. There only remains the *great question of the rate of mortality*; if the rate of mortality among officers and their wives differ essentially from that adopted in the tables, the fund will either become ultimately insolvent, and a burden on the Government, or be withholding a portion of the annuities due to the present generation, for the generations following. Upon this point we are bound to furnish the Government with the fullest information; which will be accomplished when the returns, some of which are in the course of analysis, are completed.

The importance of obtaining all the information we can respecting the mortality of officers, their wives, and children, will appear from the fact, that the question is whether a *fifth* part, a *twentieth* part, or *nothing* is to be struck from the claims of officers, as set forth in the accompanying tables.

The officers of Excise, I see, have an arrangement with the *Atlas Office*, under which the wives of officers can be insured; the premiums, from the age of 25 to 45, appear to be about 25 per cent. higher than are required under the rates of the English Table. This addition is of course made for the purposes already enumerated, and to secure adequate profits to the proprietors of the *Atlas Office*. For 10*l*. a-year premium the sum insured would be:—

At Age.	By Atlas Office to Excise Officers.	By a Fund using the English Life Table.
	£	£
25.....	433·28	553·84
30.....	390·17	486·86
35.....	348·31	424·59
45.....	267·52	311·03

The rate of mortality by the English Table is higher than that actually experienced among insured lives†; and 3 per cent. interest of

* Griffith Davies on Life Insurance; an unpublished work. A copy, presented by the author, is in the General Register Office.

† See Registrar General's Fifth Annual Report, 5vo., pp. 338—341.

money is lower than that actually realized by life offices*, so that the profits of the Atlas can scarcely be less than a fifth part of the premiums, under their arrangement with the Excise fund.

If a similar arrangement were made with the proposed Civil Service Fund, and the annual contribution remaining only 40,000*l.*, were paid to the Atlas, 8,000*l.* a-year would be profit; for 32,000*l.* a-year would probably meet all the policies they granted to officers. It is right to state that the Directors of the Atlas, after taking about 20 per cent. of the premiums as profits, make a "*gift of a half per cent.*" on the amount of premiums paid each year for Excise insurances, on donations, and on bequests of bonuses. It is not necessary to inquire here into the value of the *bonuses*, the way "profits" are shared, the manner of dealing with lapsed policies, or other matters†; the case is only mentioned as illustrative of the extent to which claims on the Civil Service Fund would be cut down, according to the practice of a respectable insurance office, of which Mr. Ansell is the judicious actuary‡.

Under any arrangement a GUARANTEE FUND must be formed, from which the deficiencies, if any are discovered at the periodical investigations, must be supplied; or out of which, if there be a surplus, safely available, additions may be granted in the shape of increased allowances to widows and orphans.

The sources from which the guarantee fund may be formed are:—

1. The contributions lapsing in the first five years of service, before an officer has any claim on the provident fund.

2. The contributions paid quarterly, and which would in the calculation only be assumed to be carried to the insurance account annually. At the close of every life *none, one, two, or three* quarterly payments, not required by the insurance fund, would go to the guarantee fund.

3. The interest of the contributions of all officers in the first five years, and interest arising from the quarterly payments. Thus, if a man contributed 20*l.* a-year, in quarterly payments, the payment of January 1st, say, would, with the three previous payments, make the annual premium then payable; the 5*l.* paid in April, the 5*l.* in July the 5*l.* in October would at simple interest give 7*s.* 6*d.* a-year for the guarantee fund, over and above the 20*l.* premium required.

4. Forfeited claims of officers dismissed the service.

* The Scottish Widows' Fund Life Assurance Society, which had 1,701,633*l.* invested in 1846, made more than 4 per cent. interest, without sustaining any loss. See Report, 1846, and other publications.

† At the time of writing I have only before me the account of this arrangement in the Tabular Statement.

‡ Analytical Table of Funds in connection with Public Departments, by Mr. Willis. The annual premium to insure 100*l.* at the age of 25 is 1·806*l.* by English Table, interest 3 per cent., and 1·619*l.* taking interest at 4 per cent.; it is 2·308*l.* by Excise Table. At the age of 30 the premiums are 2·054*l.*, and 1·855*l.* by English Table, 2·563*l.* by Atlas Table for Excise. At 30, reckoning interest at 3 per cent., the Atlas has nearly 20*l.* in every 100*l.* of the premiums for profit. The Atlas Table appears to be framed from the Old Northampton Table, interest about 3½ per cent., without additions; but it is shown in the Appendix to the Registrar General's Eighth Report that the Old Northampton Table is entirely erroneous.

5. A portion (say *a third*) of the existing claims of officers, who *resign* from other causes than ill health or infirmity(?)

6. A portion (say *a third*) of officers' claims who leave no widow or children(?)

7. Officers' claims who leave no near dependent relatives.

When the guarantee fund was placed on a secure basis, the residue might go to a benevolent fund.

It remains to determine, when all the information is obtained, whether, in addition to these sources of income, the guarantee fund should take 5 per cent. from the claims. With the best life tables that could be framed it would be scarcely necessary to take more.

It will be time enough to consider what life table should be used; and if any and what deduction should be made from the claims, for security to the fund and the Government when all the data required are arranged. In the mean time those interested will find in the accompanying tables materials for forming their own opinion on some of the most important questions connected with the constitution of the fund.

APPENDIX TO THE PAPER ON FUNDS FOR WIDOWS AND ORPHANS.

Illustrations of the different Changes which may take place in an Officer's Family, in connexion with the Fund.

It will be useful to consider some of the changes which may take place in an officer's family, and alter his relations to the fund.

This will show the various cases for which provision has to be made.

In the first place, each member of the family may be *alive* or *dead* at the end of a given time,—say a year.

For convenience of representation let there be a husband, wife, son, and daughter, and no other near relatives.

Let H	denote that the officer is alive,	H	that he is dead.
Let W	denote that his wife is alive,	W	that she is dead.
Let S	denote that his son is alive,	S	that he is dead.
Let D	denote that his daughter is alive,	D	that she is dead.

OFFICER MARRIED.

1. H W S D....The family all alive.
2. H W S D....Officer, wife, daughter, living; son dead.
3. H W S D....Officer, wife, son, living; daughter dead.
4. H W S D....Officer, wife, living; son, daughter, dead. He is childless.

OFFICER A WIDOWER.

5. H W S D....Officer, son, daughter, living; wife dead. He is a widower with two children.
6. H W S D....Officer, daughter, living; wife, son, dead. He is a widower with an only daughter.
7. H W S D....Officer, son, living; wife, daughter, dead. He is a widower with an only son.
8. H W S D....Officer living; wife, son, daughter, dead. He is a widower childless.

WIFE LEFT A WIDOW.

9. *H W S D*....Officer dead; wife, son, daughter, living. He leaves a widow and two children.
 10. *H W S D*....Officer, son, dead; wife, daughter, living. He leaves a widow and one daughter.
 11. *H W S D*....Officer, daughter, dead; wife, son, living. He leaves a widow and one son.
 12. *H W S D*....Officer, son, daughter, dead; wife, living. He leaves a widow alone.

CHILDREN ORPHANS.

13. *H W S D*....Officer, wife, dead; son, daughter, living. Two children orphans.
 14. *H W S D*....Officer, wife, son, dead; daughter living. An orphan daughter.
 15. *H W S D*....Officer, wife, daughter, dead; son living. An orphan son.
 16. *H W S D*....Officer, wife, son, daughter, dead. The family extinct.

In the first eight cases the salary and contribution to the fund would go on; but the shares of the provision for the wife and children would be subject to seven changes, or, including the first, eight.

In the last eight cases at the officer's death the contributions would cease; in four the widow would fall on the fund, three times with children, once without; in three cases children alone would fall on the fund. In seven cases charges would be left on the fund, in one case the contribution and charge would fail together.

Viewing the two facts—of life and decease only—it may be demonstrated, that, whatever the number of the family may be, 2^n will represent the number of possible changes; n being the number in the family.

$\frac{2^n}{2} - 1 = 2^{n-1} - 1$ will represent the number of ways in which the family may be left chargeable on the fund. In the previous case $2^4 = 16$, is the number of changes. $2^3 - 1 = 8 - 1 = 7$, is the number of ways in which the family may be left chargeable. Suppose the family to consist of three persons, officer, wife, son; then $2^3 = 8$ changes may take place; and in three ($n^2 - 1 = 3$) the family would be left chargeable.

H W S		H W S
H W S		H W S
H W S		H W S
H W S		H W S

If the family consist of officer and wife, the changes are $2^2 = 4$; the ways in which the family may be left chargeable $2^1 - 1 = 1$.

H W		H W
H W		H W

If the officer stand alone the changes are 2; and he can leave no charge on the fund. Changes $2^1 = 2$; $2^0 - 1 = 1 - 1 = 0$ ways in which a family may be chargeable.

H		H
---	--	---

All these changes may be obtained by the multiplication of the factors $H + W$, &c.; thus taking the family of 4:—

Number of Combinations..... }	1.	4.	6.	4.	1.
$(H+H)(W+W)\{$ $(S+S)(D+D)\} =$	H W S D +	H W S D + H W S D H W S D H W S D	H W S D + H W S D H W S D H W S D H W S D H W S D	H W S D + H W S D H W S D H W S D .	H W S D
	All may be alive in one way.	Some <i>one</i> dead, <i>three</i> living in four ways.	Some <i>two</i> dead, and <i>two</i> living in six ways.	Some <i>three</i> dead, and <i>one</i> living in four ways.	All dead in <i>one</i> way.

The number of ways in which 1, 2, 3, 4... n , may be left alive, or dead in a family of n persons, will be given by $(1+1)^n$, or by the binomial coefficients.

In a family of three we have $(1+1)^3 = 1+3+3+1$.

Number of Combinations	1.	3.	3.	1.
And $(H+H)(W+W)(S+S) =$	H W S +	H W S + H W S H W S	H W S + H W S H W S	H W S

The chances of *any one* of these changes happening are very various: thus, let 1 in 21 officers, 1 in 31 wives, 1 in 41 children die in a year,

Then it is 20 to 1 that the officer will be alive.

30 to 1 that the wife will be alive.

40 to 1 that the child will be alive.

Put therefore $H + H = 20 + 1$.

$W + W = 30 + 1$.

$S + S = 40 + 1$.

$$\begin{aligned}
 \text{Now, } (20+1)(30+1)(40+1) &= 20 \times 30 \times 40 + 20 \times 30 \times 1 \\
 &\quad + 20 \times 1 \times 1 + 1 \times 1 \times 1 \\
 &\quad + 20 \times 1 \times 40 + 1 \times 30 \times 1 \\
 &\quad + 1 \times 30 \times 40 + 1 \times 1 \times 40 \\
 &= 24,000 + 600 + 20 + 1 = 21 \times 31 \times 41 \\
 &\quad \quad \quad 800 \quad 30 \\
 &\quad \quad \quad 1,200 \quad 40
 \end{aligned}$$

$$= 24,000 + 2,600 + 90 + 1 = 26,691.$$

RULE.—To obtain these products insert 20 for H , 30 for W , 40 for S , and 1 for H , 1 for W , 1 for S , in the above formula.

$\frac{24,000}{26,691}$	$= (24,000 \text{ to } 2,691)$, the chance that officer, wife, and child will be alive.
$\frac{600}{26,691}$	$= (600 \text{ to } 26,391)$, the chance that only officer and wife will be alive; child dead.
$\frac{800}{26,691}$	$= (800 \text{ to } 26,191)$, the chance that only officer and child will be alive; wife dead.
$\frac{1,200}{26,691}$	$= (1,200 \text{ to } 25,791)$, the chance that only officer will be dead; wife and child alive.
$\frac{20}{26,691}$	$= (20 \text{ to } 26,971)$, the chance that officer only will be alive; wife and child dead.
$\frac{30}{26,691}$	$= (30 \text{ to } 26,961)$, the chance that wife only will be alive; officer and child dead.
$\frac{40}{26,691}$	$= (40 \text{ to } 26,951)$, the chance that child only will be alive; officer and wife dead.
$\frac{1}{26,691}$	$= (1 \text{ to } 26,990)$, the chance that all will be dead at the end of a year.
$\frac{2,600}{26,691}$	$=$ the chance that some <i>two only</i> will be alive.
$\frac{90}{26,691}$	$=$ the chance that some <i>one only</i> will be alive.
$\frac{26,990}{26,691}$	$=$ the chance that some <i>three, two, or one</i> will be alive.

It is 26,990 to 1 that some one of the three will live a year.

We have hitherto taken only two conditions: we may now suppose each person of the family liable to three changes of condition.

Thus the officer may be	(1) alive, (2) superannuated, (3) dead.
„ wife may be	(1) alive, (2) divorced, (3) dead.
„ son may be	(1) alive, (2) above the age of 18, (3) dead.
„ daughter may be	(1) alive, (2) above the age of 21, (3) dead.

Then, in its relations to the fund, the family may, in a given time, undergo 3^4 changes, $= 81$ changes; into which each condition of each person would enter $3^3 = 27$ times.

In all cases let a = represent the number of changes.

„ n = the number of persons in a family.

„ a^n will represent the number of possible changes.

„ a^{n-1} will represent the number of times each condition of each person enters into the a^n changes.

EXAMPLE.—In a family of six, each of which is subject to the above three changes of condition; $a^n = 3^6 = 2,187$ changes may occur. In 729 of these changes officer would be dead. In 729 of these changes officer would be superannuated, &c. &c. &c. Without going into mathematical details, these instances show some of the many contingencies that may arise in families connected with the fund.

The principal Formulæ referred to in the paper on the Civil Service Fund.

1. $\frac{D_x}{M_x} = S$ = the sum which l. in a single premium, at age x , will insure.

2. $\frac{N_x}{M_x} = \frac{N_x}{D_x - dN_x} = S$ = the sum which an annual premium of 1*l.* will insure at age *x*.

3. $\pi_x = \frac{M_x}{N_x}$ = the annual premium—commenced at age *x*—which will insure 1*l.*

4. $\frac{N_x}{D_x} = 1 + A_x$ = the present value of an annuity of 1*l.* a year on the life of a person of the age *x*; the first payment to be made at the age *x*.

5. $\frac{N_{x+1}}{D_x} = A_x$, the present value of an annuity of 1*l.*; the first payment to be made at the age *x* + 1. This is what is commonly known as the present value of a life annuity, or the year's purchase of an annuity.

6. $\frac{N_{x+n}}{D_x}$ = the present value of an annuity; first payment at the end of *n* years.

7. $\frac{N_x - N_{18}}{D_x}$ = the present value of an annuity of 1*l.*, on the life of a boy aged *x*; for the next 18 — *x* years. The first payment to be made at age *x*, and the last at 17.

8. $\frac{N_y - N_{21}}{D_y}$ = value of a similar annuity on a female life; last payment at the age of 20.

9. $\frac{N_{x,y}}{D_{x,y}} = 1 + A_{x,y}$ = the present value of an annuity of 1*l.*, on the joint life of a male, age *x*, and of a female, age *y*, payable at the beginning of every year that both live.

10. $\frac{N_y}{D_y} - \frac{N_{x,y}}{D_{x,y}} = A_y - A_{x,y}$ = the present value of an annuity of 1*l.*; the first payment to the wife, age *y*, to be made at the end of the year in which the husband, now of the age *x*, shall die.

11. $\frac{1 + A_{x,y}}{A_y - A_{x,y}}$ = the survivorship annuity which 1*l.* a year premium, paid during the joint lives of a male, age *x*, and a female, age *y*, will provide for the female after the death of the male.

12. $S \times \pi_x = S \times \frac{M_x}{N_x}$ = the annual premium which, commenced at age *x*, will insure *S*l.**

13. $S \times \frac{M_x}{N_x} \times \frac{1 + A_{x,y}}{A_y - A_{x,y}}$ = the survivorship annuity which *S*l.**, insured at the death of the husband, now age *x*, will provide for the widow. If she die first, the annual premium $S \times \frac{M_x}{N_x}$ is set free.

It is evident that if the annual premium $= \frac{M_x}{N_x} \times 1l.$ will insure $1l.$
 $S \times \pi_x = S \times \frac{M_x}{N_x}$ is the premium which will insure $Sl.$; or, in other words, the annual premium, of $Sl. \times \frac{M_x}{N_x}$, is equal in value to the sum $Sl.$ paid at death; but $1l.$ a year, commenced at age x , and paid during the joint lives will provide a survivorship annuity of $\frac{1 + A_{x,y}}{A_y - A_{x,y}} \times 1l.$, and therefore the annual premium, $S \times \pi_x$, commenced at age x , will provide the survivorship annuity of $S \times \pi_x \times \frac{1 + A_{x,y}}{A_y - A_{x,y}} \times 1l.$

$$14. \quad S \times \pi_x \times \frac{1 + A_x}{A_y - A_{x,y}} = S \times \frac{M_x}{N_x} \times \frac{\frac{N_x}{D_x}}{A_y - A_{x,y}}$$

$$= S \times \frac{M_x}{D_x} \times \frac{1}{A_y - A_{x,y}}.$$

This result is, in itself, sufficiently evident; $S \times \frac{M_x}{D_x}$ is the *present value* of $Sl.$ insured at death by the annual premiums $\pi_x \times S$; and as $A_y - A_{x,y}$ will insure a survivorship annuity of $1l.$

$(A_y - A_{x,y}) : 1 :: S \times \frac{M_x}{D_x} : S \times \frac{M_x}{D_x} \times \frac{1}{A_y - A_{x,y}}$
 $=$ the survivorship annuity which $Sl.$ insured will secure. The annual premiums, by which it is provided, commenced at age x , must be continued to the end of officer's life.

15. Let C = annual contribution at age x , to be continued to end of life : then if an officer discontinue his contribution, from superannuation or other cause:

$S - C \frac{N_x}{M_x}$ = the amount of his claim after the *discontinuance of his annual contribution*, due at the age x , and at every future year of his life.

As $\frac{N_x}{M_x}$ is the mean amount at death of $1l.$ a year, laid up at interest during the life of persons first contributing at age x ;

$\frac{N_x}{M_x} C$, is the amount to which a person contributing C pounds has an equitable claim. The contribution ceasing at x , then $\frac{N_x}{M_x} C$ must be deducted from his *previous claim*.

16. Let ${}_a n_y$ = an annuity to a female, age y , after the death of a

male, age x , provided by a premium $= \frac{1}{2} C$, during joint lives; then contribution ceasing, the survivorship annuity is diminished: thus it becomes $= a_x - \frac{1}{2} C \times \frac{1 + A_{x,y}}{A_y - A_{x,y}}$.

Investigation of the effect of discontinuance of contribution on survivorship annuity.

The husband was only bound to continue the contribution during the joint lives, to entitle his widow to the annuity a_x ; if he discontinue his contribution (C) at the age x , it is evident that a_x will be reduced by $\frac{1}{2} C \frac{(1 + A_{(x,y)})}{A_y - A_{(x,y)}}$. The annuity to which she will be entitled after the discontinuance of the contribution $\frac{1}{2} C$

$$\text{will therefore be} = a_x - \frac{1}{2} C \frac{(1 + A_{(x,y)})}{A_y - A_{(x,y)}}$$

EXAMPLE.—Contributor discontinuing, who has a claim for an insurance of 399·179*l.*, and an annuity, besides, of 33·225*l.* to his widow; his contribution being 15*l.* a year, half of which is required for the survivorship annuity, half for insurance.

Discontinuing at Age	The Insurance Account falls to	His Widow's Survivorship Annuity falls to
	£ £ £	£ £ £
50	399·179 - 193·650 = 205·529	33·225 - 23·949 = 9·276
65	399·179 - 95·535 = 303·644	33·225 - 16·292 = 16·933
		The discontinuance of the contribution at the advanced age of 65 strikes off 16·292 <i>l.</i> from the survivorship annuity.

The Uniform Annuity to Children.

Let A_v, A_w, A_u, A_y, A_z , represent the *present value* of an annuity of 1*l.* on the lives of persons of the respective ages v, w, u, y, z ,
 S

Then $\frac{S}{A_v + A_w + A_u + A_y + A_z} = x$ = the uniform annuity to each

which S *l.* will provide; for $A_v + A_w + A_u + A_y + A_z : S :: 1 : x$

Assume that the sum of the uniform annuities is $= xA_v + xA_w + xA_u + xA_y + xA_z = S$ from the conditions of the question. And
 S

$$x(A_v + A_w + A_u + A_y + A_z) = S \therefore x = \frac{S}{A_v + A_w + A_u + A_y + A_z}$$

If the annuities are *temporary* or *deferred* by substituting the present values of the same for the values of annuities for life, these modifications of the question will be solved. If the annuity is to be paid at the beginning, instead of the end of the year, the same principle will evidently hold.

The Capital Sums and the Survivorship Annuities to be insured by the Annual payment of 1l. 50l. and 25l. to the "Civil Service Provident Fund."

Age.	Sums insured by the Annual Life Premiums.			Survivorship Annuity to Widow, insured by Annual Premiums payable during Life of Husband and of Wife—of equal Ages.		
	PREMIUMS.					
	£1	500	250	£1	500	250
	Sums Insured.			Survivorship Annuities.		
15....	70·511	35·255	17·628	4·9491	2·4745	1·2373
16....	68·880	34·440	17·220	4·8866	2·4433	1·2216
17....	67·275	33·638	16·819	4·8255	2·4127	1·2063
18....	65·700	32·850	16·425	4·7657	2·3829	1·1914
19....	64·154	32·077	16·039	4·7071	2·3536	1·1768
20....	62·631	31·315	15·658	4·6498	2·3249	1·1625
21....	61·135	30·568	15·284	4·5937	2·2968	1·1484
22....	59·660	29·890	14·915	4·5388	2·2694	1·1347
23....	58·215	29·107	14·554	4·4850	2·2425	1·1212
24....	56·785	28·393	14·196	4·4323	2·2162	1·1081
25....	55·384	27·692	13·846	4·3808	2·1904	1·0952
26....	54·006	27·003	13·502	4·3301	2·1650	1·0825
27....	52·645	26·322	13·161	4·2806	2·1403	1·0702
28....	51·306	25·653	12·826	4·2319	2·1160	1·0580
29....	49·988	24·994	12·497	4·1838	2·0919	1·0459
30....	48·686	24·343	12·172	4·1372	2·0686	1·0343
31....	47·408	23·704	11·852	4·0910	2·0455	1·0228
32....	46·144	23·072	11·536	4·0456	2·0228	1·0114
33....	44·899	22·450	11·225	4·0008	2·0004	1·0002
34....	43·672	21·836	10·918	3·9566	1·9783	·9891
35....	42·459	21·229	10·615	3·9128	1·9564	·9782
36....	41·265	20·633	10·316	3·8693	1·9346	·9673
37....	40·084	20·042	10·021	3·8261	1·9131	·9565
38....	38·916	19·458	9·729	3·7830	1·8915	·9458
39....	37·763	18·881	9·441	3·7399	1·8699	·9350
40....	36·624	18·312	9·156	3·6965	1·8483	·9241
41....	35·497	17·749	8·874	3·6527	1·8263	·9132
42....	34·383	17·191	8·596	3·6083	1·8042	·9021
43....	33·280	16·640	8·320	3·5631	1·7815	·8908
44....	32·186	16·093	8·047	3·5167	1·7584	·8792
45....	31·103	15·552	7·776	3·4686	1·7343	·8672
46....	30·029	15·014	7·507	3·4190	1·7095	·8548
47....	28·964	14·482	7·241	3·3671	1·6835	·8418
48....	27·908	13·954	6·977	3·3124	1·6562	·8281
49....	26·860	13·430	6·715	3·2546	1·6273	·8136
50....	25·819	12·910	6·455	3·1931	1·5966	·7983
51....	24·782	12·391	6·195	3·1272	1·5636	·7818
52....	23·752	11·876	5·938	3·0561	1·5280	·7640
53....	22·728	11·364	5·682	2·9794	1·4897	·7449
54....	21·707	10·853	5·427	2·8960	1·4480	·7240
55....	20·691	10·346	5·173	2·8052	1·4026	·7013
56....	19·735	9·867	4·933	2·7246	1·3623	·6811
57....	18·832	9·416	4·708	2·6528	1·3264	·6632
58....	17·961	8·981	4·490	2·5832	1·2916	·6458
59....	17·122	8·561	4·280	2·5163	1·2582	·6291
60....	16·315	8·157	4·079	2·4521	1·2260	·6130
61....	15·540	7·770	3·885	2·3907	1·1954	·5977
62....	14·795	7·398	3·699	2·3320	1·1660	·5830
63....	14·081	7·040	3·520	2·2761	1·1380	·5690
64....	13·396	6·698	3·349	2·2229	1·1115	·5557
65....	12·738	6·369	3·184	2·1723	1·0861	·5431
66....	12·108	6·054	3·027	2·1245	1·0623	·5311
67....	11·504	5·752	2·876	2·0791	1·0396	·5198
68....	10·927	5·404	2·732	2·0362	1·0181	·5091
69....	10·374	5·187	2·594	1·9958	·9979	·4989
70....	9·846	4·923	2·461	1·9578	·9789	·4895

The preceding Table shows, in pounds and decimals of a pound, the Sums which 1*l.*, 10*s.*, and 5*s.* a year, commenced at any age, from 15 to 70, will insure, according to the rates of the English Life Table. It also shows the Survivorship Annuities which the same premium will provide for a wife of the same age as her husband: if she be younger, the annuities will be less; if older, they will be greater than those in the table.

Table for computing the Sums required to provide "Orphans' Annuities" from the "Civil Service Provident Fund."

Present value of Annuities of 1*l.* payable at the BEGINNING of each year of age until the Male is 18, and the Female 21 years of age.

	Males.	Females.
0	10·475	11·975
1	11·610	13·032
2	11·663	13·190
3	11·368	12·994
4	10·947	12·650
5	10·432	12·214
6	9·857	11·711
7	9·231	11·156
8	8·560	10·558
9	7·847	9·924
10	7·097	9·256
11	6·311	8·556
12	5·498	7·824
13	4·657	7·064
14	3·790	6·282
15	2·893	5·476
16	1·964	4·646
17	1·000	3·784
18	2·890
19	1·963
20	1·000
21		

Example of the Widows' Annuity, and of the Capital Sum for the support of Orphans, to be insured by the contribution to the "Civil Service Provident Fund" of the Assessments at present levied on Public Incomes, under Act 4 & 5 Will. IV., cap. 4.

Age.	Years of Service.	Salary.	Annual Contribution.	Total Amount Contributed.	Aggregate Sum insured by Contributions continued to last year of life.	Insurance Account.	Annuity Account.
		£	£	£	£	Sum to provide for Children.	Annuity for Widow.
17½	0					Officer has no claim until 5th Payment has been made: the 37-3000. is the sum insured by the single premium 13-6250.	
18.....	1	45	1-125	1-125	..		
19.....	2	90	2-250	3-375	..		
20.....	3	90	2-250	5-625	..		
21.....	4	100	2-500	8-125	..		
22.....	5	110	5-500	13-625	37-300		
23.....	6	120	6-000	19-625	386-590	386-590	..
						Officer married to wife of his own age.	
24.....	7	130	6-500	26-125	414-983	207-492	16-196
25.....	8	140	7-000	33-125	442-675	221-333	17-291
26.....	9	150	7-500	40-625	469-678	234-839	18-374
27.....	10	160	8-000	48-625	496-001	248-001	19-444
28.....	11	170	8-500	57-125	521-654	260-827	20-502
29.....	12	180	9-000	66-125	546-648	273-324	21-548
30.....	13	190	9-500	75-625	570-991	285-496	22-582
31.....	14	200	10-000	85-625	594-695	297-348	23-605
32.....	15	210	10-500	96-125	617-767	308-884	24-616
33.....	16	220	11-000	107-125	640-217	320-109	25-616
34.....	17	230	11-500	118-625	662-053	331-027	26-605
35.....	18	240	12-000	130-625	683-283	341-642	27-583
36.....	19	250	12-500	143-125	703-916	351-958	28-550
37.....	20	260	13-000	156-125	723-968	361-979	29-507
38.....	21	270	13-500	169-625	743-416	371-708	30-453
39.....	22	280	14-000	183-625	762-298	381-149	31-388
40.....	23	290	14-500	198-125	780-610	390-305	32-312
41.....	24	300	15-000	213-125	798-359	399-179	33-225
42.....	25	300	15-000	228-125	798-359	399-179	33-225
43.....	26	300	15-000	243-125	798-359	670-942	WIFE DE-CEASED.
44.....	27	300	15-000	258-125	798-359		
45.....	28	300	15-000	273-125	798-359	The Insurance Account remains £670-942.	If Wife survive and Officer is superannuated, his Contribution ceasing after age 64, and before age 65, his widow will be entitled to the annuity below.
46.....	29	300	15-000	288-125	798-359		
47.....	30	300	15-000	303-125	798-359		
48.....	31	300	15-000	318-125	798-359		
49.....	32	300	15-000	333-125	798-359		
50.....	33	300	15-000	348-125	798-359		
51.....	34	300	15-000	363-125	798-359		
52.....	35	300	15-000	378-125	798-359		
53.....	36	300	15-000	393-125	798-359		
54.....	37	300	15-000	408-125	798-359		
55.....	38	300	15-000	423-125	798-359		
56.....	39	300	15-000	438-125	798-359		
57.....	40	300	15-000	453-125	798-359		
58.....	41	300	15-000	468-125	798-359		
59.....	42	300	15-000	483-125	798-359		
60.....	43	300	15-000	498-125	798-359		
61.....	44	300	15-000	513-125	798-359		
62.....	45	300	15-000	528-125	798-359		
63.....	46	300	15-000	543-125	798-359		
64.....	47	300	15-000	558-125	798-359		
65.....	48	300	15-000	573-125	798-359		
65.....	If Officer should be Superannuated and discontinue Contribution at 65, the Sums insured would be reduced			..	607-239	479-873	16-933
70.....	53	300	15-000	648-125	798-359		
75.....	58	300	15-000	723-125	798-359		
80.....	63	300	15-000	798-125	798-359		
85.....	68	300	15-000	873-125	798-359		
90.....	73	300	15-000	948-125	798-359		

NOTE.—The Officer entering in any year at the age of 17, and under 18, is assumed to be 18 on January 1st of the year following: the contributions up to that date are carried to his account, and considered as his first payment to the fund.

Examples of the Annuities which may be Provided on the Second Plan for Widows and Orphans by the Conversion into a "Provident Fund" of the Assessments levied under the Act 4 & 5 William IV., cap. 24.

No. of Officers.	In each case the Officer is assumed to marry at the Age of 25 a Wife of his own age.											
	First Officer dies at 28.				Second Officer dies at 45.				Third Officer dies at 57.			
	Age and Rate of Salary on entering the Service.		Widows.		Number and Ages of Children left.		Widows.		Number and Ages of Children left.		Widows.	
	Age.	Salary.	Total Amount of Insurance.		Boys.	Girls.	Total Amount of Insurance.	Boys.	Girls.	Total Amount of Insurance.	Boys.	Girls.
			Survivorship Annuity Insured by half the Contributions.	Increased Annuity if no Children are living under Age.			Survivorship Annuity Insured by half the Contributions.			Survivorship Annuity Insured by half the Contributions.		
3 in Customs ..	Yrs.	£	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
	16	80	200 6 16	24 7 0	39 5 0	0	23 7 0	7 0	0	16 14 72	2 0	137 12 0
3 in Customs ..	19	90	140 423	16 16 0	27 0 0	0	16 0 0	180	513	20 16 0	36 6 0	0
	16	90	250 7 56	31 3 0	50 2 0	0	29 15 0	15 0	0	450 1204	50 12 0	99 8 0
3 in Ordnance ..	16	90	90 169	6 13 0	10 15 0	0	6 8 0	220	542	23 16 0	40 3 0	0
	16	90	90 169	6 13 0	10 15 0	0	6 8 0	220	542	23 16 0	40 3 0	0

This Table may be read thus:—One of the Officers entering the Customs at 16 years of age, on a salary of 80*l.* per annum, marries at the age of 25 a wife of his own age—he dies at the age 45, enjoying at the time of death a salary of 700*l.* per annum. The aggregate sum insured, and due at the end of the year in which he dies, is 1,614*l.*; but, half the sum of his contributions has been sunk to provide a Survivorship Annuity to his Widow, of 72*l.* 2*s.* 0*d.*; thus, leaving 807*l.* to provide annuities for his children. If, however, no children are living under age, the annuity to his widow will be increased to 1204*l.* 16*s.* 0*d.*. He has, say, three children, namely, 2 boys, 10 and 8 years of age, a girl 17; each of them will enjoy an annuity of 41*l.* 10*s.* 0*d.*, until the boys attain the age of 18—the girl 21.

Example of Three Plans upon which Annuities to Widows may be granted.

Year.	Precise Age.	Salary.	Contribution.	Aggregate Sum Insured by Contributions continued until year of death.	Half Aggregate Sum Insured: (for Wife, if no Annuity be granted until Officer's death.)	Half Sum Insured: (to provide Annuities) for Children.	Annuity to Widow if Officer's decease take place previous to that of his Wife, and at the Ages stated in 1st Column.		
	1	2	3	4	5	6	Annuity that half the Sum Insured will provide on Widow's Life	Widow's Survivorship Annuity at decease of Husband.	First Widow's Survivorship Annuity, the Premiums if Wife die first not going to Insurance, but required to the end of Husband's life.
	£	£	£	£	£	£	£	£	£
Jan. 1st.	17½								
1850	18	45·000	1·125	Officer has no claim until 5th Payment has been made. He is then entitled to the Sums above the black line.			If the Wife die first the Sums in black type are carried to the Insurance Account, column 6, at the beginning of the next year.		
1851	19	90·000	2·250						
1852	20	90·000	2·250						
1853	21	100·000	2·50						
1854	22	110·000	5·5						
				37·300	18·650	18·650			
1855	23	120·000	6·0	386·590	193·295	193·295	(1st Plan.)	(2nd Plan.)	(3rd Plan.)
1856	24	130·000	6·5	414·983	207·492	207·492	10·036	16·196	19·695
1857	25	140·000	7·0	442·675	221·338	221·338	10·803	17·291	20·922
1858	26	150·000	7·5	469·678	234·840	234·840	221·338	210·84	Nil.
1859	27	160·000	8·0	496·001	248·001	248·001	12·333	19·444	23·535
1860	28	170·000	8·5	521·654	260·827	260·827	248·00	225·95	Nil.
1861	29	180·000	9·0	546·648	273·324	273·324	13·865	21·548	26·095
1862	30	190·000	19·5	570·991	285·496	285·496	14·633	22·582	27·356
1863	31	200·000	10·0	594·695	297·348	297·348	15·404	23·005	28·604
1864	32	210·000	10·5	617·767	308·884	308·884	16·179	24·616	29·840
1865	33	220·000	11·0	640·217	320·109	320·109	16·959	25·616	31·064
1866	34	230·000	11·5	662·053	331·027	331·027	17·745	26·606	32·276
1867	35	240·000	12·0	683·283	341·642	341·642	18·538	27·583	33·476
1868	36	250·000	12·5	703·916	351·958	351·958	19·339	28·550	34·664
1869	37	260·000	13·0	723·968	361·979	361·979	20·150	29·507	35·841
1870	38	270·000	13·5	743·416	371·708	371·708	20·973	30·453	37·008
1871	39	280·000	14·0	762·298	381·149	381·149	21·810	31·388	38·161
1872	40	290·000	14·5	780·610	390·305	390·305	22·661	32·312	39·303
1873	41	300·000	15·0	798·359	399·179	399·179	23·530	33·225	40·433
1874	42	300·000	15·0	798·359	399·179	399·179	399·18	271·76	Nil.
1875	43	300·000	15·0	798·359	399·179	399·179	24·301	33·225	40·433
1876	44	300·000	15·0	798·359	399·179	399·179	24·721	33·225	40·433
1877	45	300·000	15·0	798·359	399·179	399·179	25·169	33·225	40·433
1878	46	300·000	15·0	798·359	399·179	399·179	25·647	33·225	40·433
1879	47	300·000	15·0	798·359	399·179	399·179	26·158	33·225	40·433
1880	48	300·000	15·0	798·359	399·179	399·179	26·706	33·225	40·433
1881	49	300·000	15·0	798·359	399·179	399·179	399·18	212·32	Nil.
1882	50	300·000	15·0	798·359	399·179	399·179	399·18	204·07	Nil.
1883	51	300·000	15·0	798·359	399·179	399·179	28·616	33·225	40·433
1884	52	300·000	15·0	798·359	399·179	399·179	29·362	33·225	40·433
1885	53	300·000	15·0	798·359	399·179	399·179	30·174	33·225	40·433
1886	54	300·000	15·0	798·359	399·179	399·179	31·063	33·225	40·433
1887	55	If the Salary be the same all the other elements remain unchanged.				399·179	32·037	33·225	40·433
1888	56					399·179	33·045	33·225	40·433
1889	57					399·179	34·119	33·225	40·433
1890	58					399·179	35·263	33·225	40·433
1891	59					399·179	36·482	33·225	40·433
1892	60					399·179	399·18	128·96	Nil.
1893	61					399·179	39·166	33·225	40·433
1894	62					399·179	40·644	33·225	40·433
1895	63					399·179	42·223	33·225	40·433
1896	64					399·179	43·910	33·225	40·433
1897	65					399·179	399·18	100·68	Nil.
1902	70					399·179	56·903	33·225	40·433
1907	75					399·179	73·033	33·225	40·433
1912	80					399·179	96·768	33·225	40·433